Intermode-coupling modulation in the fermion-boson model: heating effects in the BCS regime

J. Plata

Departamento de Física, Universidad de La Laguna,

La Laguna E38204, Tenerife, Spain.

Abstract

Heating induced by an oscillating modulation of the interaction strength in an atomic Fermion pair condensate is analyzed. The coupled fermion-boson model, generalized by incorporating a time-dependent intermode coupling through a magnetic Feshbach resonance, is applied. The dynamics is analytically characterized in a perturbative scheme. The results account for experimental findings which have uncovered a damped and delayed response of the condensate to the modulation. The delay is due to the variation of the quasiparticle energies and the subsequent relaxation of the condensate. The detected damping results from the excitations induced by a nonadiabatic modulation: for driving frequencies larger than twice the pairing gap, quasiparticles are generated, and, consequently, heating sets in.

PACS numbers: 03.75Ss, 05.30.Fk

I.   INTRODUCTION

The realization of the crossover from a molecular Bose-Einstein condensate (BEC) to a Bardeen-Cooper-Schrieffer (BCS) superfluid of atom pairs [1–12] has opened the way to a variety of experiments on fundamental effects in quantum statistics and many-body physics. Essential to the versatility of this scenario has been the possibility of varying the interaction strength via a Feshbach resonance (FR), which has allowed the characterization of the effects under controlled conditions. In parallel, the need of explaining emergent phenomenology has brought about active theoretical work in the field. Recent research has dealt with the role of thermal fluctuations and the characterization of nonequilibrium situations [13]. In this line, here, we extend previous theoretical work on the effects of an oscillating modulation of the interaction strength in a two-component Fermi gas of atoms [14]. Our objective is to complete the analysis of the experiments of Ref. [15]. In them, a gas of ultracold ^6Li atoms was prepared in the BCS regime through a magnetic FR, specifically, the (broad) FR at 834 G between the two lowest hyperfine states. The application of a sinusoidal modulation of the magnetic field was shown to lead the condensate fraction to oscillate with the driving frequency. Moreover, the oscillations were found to be damped and delayed with respect to the modulation, the damping time being much longer than the driving period. In previous work [14], the focus was put on the mechanism responsible for the delay: the modulation was shown to drive the system to an out-of-equilibrium situation, the deferred response being rooted in the finite relaxation time of the condensate. Here, we will concentrate on understanding the decay of the condensate fraction. To this end, we set up a framework where, through a partial analytical characterization of the dynamics, the origin of the damping processes can be identified. The decay of the oscillations will be linked to heating induced by the generation of quasiparticles. Nonadiabaticity, (on the gap time scale), will appear as a crucial component of the excitation process. Our scheme will allow us to trace the differential aspects of the mechanisms responsible for heating and delay in the system response.

The outline of the paper is as follows. In Section II, we generalize the coupled fermion-boson model [16–21] by incorporating a time-dependent intermode detuning, or, equivalently, a time-dependent coupling. To deal with this variation of the basic model, a perturbative scheme based on the Hartree-Fock-Bogoliubov (HFB) description is developed. In Section

2

III, the relaxation of the condensate and the associated delayed response to the modulation are tackled. The heating effects are evaluated in Section IV. In Section V, our conclusions are summarized.

II. THE FERMION-BOSON MODEL WITH A MODULATED COUPLING STRENGTH

Our system consists of a gas of ultracold fermionic atoms with two hyperfine states coupled to a molecular two-particle state through a magnetic FR. To describe it, we apply the coupled fermion-boson model [16, 18–20]. The grand-canonical Hamiltonian reads

$$
\begin{aligned}
H - \mu N = &\sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} a_{\mathbf{k},\sigma}^{\dagger} a_{\mathbf{k},\sigma} + V_{int} \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} a_{\frac{\mathbf{q}}{2}+\mathbf{k},\uparrow}^{\dagger} a_{\frac{\mathbf{q}}{2}-\mathbf{k},\downarrow}^{\dagger} a_{\frac{\mathbf{q}}{2}-\mathbf{k}',\downarrow} a_{\frac{\mathbf{q}}{2}+\mathbf{k}',\uparrow} + \\
&\sum_{\mathbf{q}} \left(\varepsilon_{\mathbf{q}}^{m} + \hbar\nu_0 \right) b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}} + g \sum_{\mathbf{q},\mathbf{k}} \left(b_{\mathbf{q}} a_{\frac{\mathbf{q}}{2}+\mathbf{k},\uparrow}^{\dagger} a_{\frac{\mathbf{q}}{2}-\mathbf{k},\downarrow}^{\dagger} + \text{h.c.} \right)
\end{aligned}
\tag{1}
$$

where μ is the chemical potential, N is the total number of atoms, $a_{\mathbf{k},\sigma}^{\dagger}$ $(a_{\mathbf{k},\sigma})$ denotes a fermionic creation (annihilation) operator of an atom with momentum $\mathbf{k}$ and spin σ, $(\sigma \in \{\uparrow,\downarrow\})$, and $b_{\mathbf{q}}^{\dagger}$ $(b_{\mathbf{q}})$ is a bosonic operator that creates (destroys) a molecule with momentum $\mathbf{q}$. It is assumed that the two hyperfine states are equally populated. $\varepsilon_{\mathbf{k}} = \hbar^2 \mathbf{k}^2/2m - \mu$ and $\varepsilon_{\mathbf{q}}^{m} = \hbar^2 q^2/4m - 2\mu$ are the free dispersion relations for fermions and bosons, respectively. $V_{int}(<0)$ characterizes the binary attractive interaction potential between fermions. Additionally, g represents the FR coupling between the closed and the open channel states, and ν_0 is the detuning of the boson resonance state from the collision continuum. (We stress that the considered approach is actually a realization of a general two-channel model to the particular context of Fermion-pair production.)

Initially, the system is at equilibrium at a finite temperature T. In that situation, a sinusoidal modulation of the detuning from the FR is applied. Correspondingly, ν_0 is replaced in Eq. (1) by $\nu(t) = \nu_0 + A \sin \omega_p t$. It is assumed that V_{int}, which corresponds to the pairing interaction resulting from nonresonant processes, is not affected by the applied driving field. Through the unitary transformation $U(t) = e^{i\frac{A}{\omega_p} \cos \omega_p t \left(\sum_{\mathbf{q}} b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}} + \frac{1}{2} \sum_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma}^{\dagger} a_{\mathbf{k},\sigma} \right)}$, the Hamiltonian, transformed as $H' = U^{\dagger} H U - i\hbar U^{\dagger} \dot{U}$, is converted into

$$H' - \mu N = \sum_{\mathbf{k},\sigma} \left(\varepsilon_{\mathbf{k}} - \hbar \frac{A}{2} \sin(\omega_p t) \right) a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k},\sigma} + V_{int} \sum_{\mathbf{q},\mathbf{k},\mathbf{k'}} a_{\frac{\mathbf{q}}{2}+\mathbf{k},\uparrow}^\dagger a_{\frac{\mathbf{q}}{2}-\mathbf{k},\downarrow}^\dagger a_{\frac{\mathbf{q}}{2}-\mathbf{k'},\downarrow} a_{\frac{\mathbf{q}}{2}+\mathbf{k'},\uparrow} +$$

$$\sum_{\mathbf{q}} (\varepsilon_{\mathbf{q}}^m + \hbar\nu_0) b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + \left(g \sum_{\mathbf{q},\mathbf{k}} b_{\mathbf{q}} a_{\frac{\mathbf{q}}{2}+\mathbf{k},\uparrow}^\dagger a_{\frac{\mathbf{q}}{2}-\mathbf{k},\downarrow}^\dagger + \text{h.c.} \right). \tag{2}$$

(In obtaining the above expression we have made use of the partial result $U^\dagger H U = H$, which derives from the commutation relation of H with the total number of fermions and the cancellation of the introduced time dependence in the interaction term.) (In Ref. [14], an alternative approach was implemented by transferring the time variation in ν to the intermode coupling via a different unitary transformation. Note, that, as shown in Ref. [14], the time dependence of the coupling term prevents the one-mode reduction, applicable to the undriven dynamics for broad resonances.) An approximate description of the dynamics resulting from the modulation can be obtained through the following perturbative scheme. The complete Hamiltonian is split as $H' - \mu N \simeq H_0 + H_{per}$, where the unperturbed Hamiltonian has the form given by Eq. (1), i.e., $H_0 = H - \mu N$; and, the perturbation reads

$$H_{per} = -\frac{\hbar}{2} A \sin(\omega_p t) \sum_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k},\sigma}.$$

(We consider that the modulation amplitude is sufficiently small for the perturbative scheme to be valid. It is assumed that the system, initially in the BCS side, stays in that regime during the whole process. Hence, the BEC side is not reached and neither is attained the unitary limit. Later on, we will precisely define the range of applicability of our approach.)

A.   The zero-order Hamiltonian

To describe the unperturbed system, we follow the standard HFB approach [18, 20]. Accordingly, we introduce first three mean fields: $n_0 \equiv \sum_{\mathbf{k}} \left\langle a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k},\sigma} \right\rangle$ for the spin density, $\Delta_0 \equiv |V_{int}| \sum_{\mathbf{k}} \langle a_{-\mathbf{k},\downarrow} a_{\mathbf{k},\uparrow} \rangle$ for the pairing field, and $\phi_{m,0} \equiv \langle b_{\mathbf{q}=\mathbf{0}} \rangle$ for the boson field. (We take $\mathbf{q} = \mathbf{0}$ as we focus on the condensed molecular field.) Through the incorporation of those mean fields, the zero-order Hamiltonian, which describes the unmodulated system, is rewritten in the form

$$H_0 = \sum_{\mathbf{k},\sigma} V_k a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k},\sigma} - \sum_{\mathbf{k}} (\tilde{\Delta}_0 a_{\mathbf{k},\uparrow}^\dagger a_{-\mathbf{k},\downarrow}^\dagger + \text{h.c.}),$$

which corresponds to an effective BCS model with mode energy $V_k \equiv \varepsilon_{\mathbf{k}} + V_{int} n_0$ and gap $\tilde{\Delta}_0 \equiv \Delta_0 - g\phi_{m,0}$. The mean-field description includes also the equation for the evolution of the boson mode, namely,

$$i\hbar \frac{d\phi_{m,0}}{dt} = (\nu_0 - 2\mu)\phi_{m,0} + \frac{g}{|V_{int}|}\Delta_0. \tag{3}$$

H_0 is straightforwardly diagonalized. By applying the Bogoliubov transformation (BT) defined by the fermionic operators $c_{\mathbf{k},\uparrow} = \cos\theta_k a_{\mathbf{k},\uparrow} - \sin\theta_k a_{-\mathbf{k},\downarrow}^\dagger$ and $c_{-\mathbf{k},\downarrow}^\dagger = \sin\theta_k a_{\mathbf{k},\uparrow} + \cos\theta_k a_{-\mathbf{k},\downarrow}^\dagger$, where θ_k is given by $\tan(2\theta_k) = \left|\tilde{\Delta}_0\right|/V_k$ [18, 20], we find

$$H_0 = \sum_{\mathbf{k}} E_{k,0}(c_{\mathbf{k},\uparrow}^\dagger c_{\mathbf{k},\uparrow} + c_{\mathbf{k},\downarrow}^\dagger c_{\mathbf{k},\downarrow}) + \text{constant}.$$

The operator $c_{\mathbf{k},\uparrow}^\dagger$ ($c_{\mathbf{k},\uparrow}$) creates (annihilates) a quasi-particle excitation with momentum $\mathbf{k}$ and spin $\uparrow$. The associated excitation energies are

$$E_{k,0} = \sqrt{V_k^2 + \tilde{\Delta}_0^2} = \sqrt{(\hbar^2 k^2/2m - \mu + V_{int} n_0)^2 + \tilde{\Delta}_0^2} \tag{4}$$

The BCS state $|\Psi_{BCS}\rangle$ is the effective vacuum state of this Hamiltonian, i.e., $c_{\mathbf{k},\uparrow}|\Psi_{BCS}\rangle = c_{\mathbf{k},\downarrow}|\Psi_{BCS}\rangle = 0$; in the previous representation, it is given by $|\Psi_{BCS}\rangle = \prod_{\mathbf{k}} \left(\cos\theta_k + \sin\theta_k a_{\mathbf{k},\uparrow}^\dagger a_{-\mathbf{k},\downarrow}^\dagger\right)|0\rangle$ [12]. Note that the excitation gap $\tilde{\Delta}_0$ combines the mean pairing field Δ_0 and the equilibrium molecular field $\phi_{m,0}$. Δ_0 is obtained from the BCS equation

$$\Delta_0 = \frac{|V_{int}|}{2}\sum_{\mathbf{k}}[2f_k^{eq}(0) - 1]\sin(2\theta_k), \tag{5}$$

where $\{f_k^{eq}(0)\}$ are the initial populations of the quasiparticle states, which are given by the Fermi distribution function, i.e., $f_k^{eq}(0) = 1/(1 + e^{E_{k,0}/k_B T})$, since thermal equilibrium is assumed for the system before the application of the magnetic modulation. Moreover, $\phi_{m,0}$ is obtained as the stationary solution to Eq. (3), namely,

$$\phi_{m,0} = \frac{g\Delta_0}{|V_{int}|(2\mu - \nu_0)}. \tag{6}$$

(Note that a selfconsistent procedure is required to obtain Δ_0 and $\phi_{m,0}$.)

B. The perturbation

We turn now to analyze the effect of the perturbation. The introduction of H_{per} in the HFB approach implies dealing with changes in the mean fields, which now become $n(t) = n_0 + \delta n(t)$, $\Delta(t) = \Delta_0 + \delta\Delta(t)$, and $\phi_m(t) = \phi_{m,0} + \delta\phi_m(t)$. Our objective is solving for the perturbation-induced increments of those fields. In particular, we will focus on explaining the damping and delay of $\delta\Delta(t)$ observed in the experiments. Let us see that a first-order approximation to that behavior can be obtained simply by incorporating H_{per} into the HFB approach defined by the unperturbed mean fields. Specifically, we apply the previously defined BT to the perturbation Hamiltonian, which, as a result, is written as $H_{per} = H_{evar} + H_{coup}$, where

$$H_{evar} = -\frac{\hbar A}{2} \sin(\omega_p t) \sum_{\mathbf{k}} \cos(2\theta_k) \left(c_{\mathbf{k},\uparrow}^\dagger c_{\mathbf{k},\uparrow} + c_{\mathbf{k},\downarrow}^\dagger c_{\mathbf{k},\downarrow} \right), \qquad (7)$$

$$H_{coup} = -\frac{\hbar A}{2} \sin(\omega_p t) \sum_{\mathbf{k}} \sin(2\theta_k) \left(c_{\mathbf{k},\uparrow}^\dagger c_{-\mathbf{k},\downarrow}^\dagger + \text{h.c.} \right). \qquad (8)$$

From the forms of H_{evar} and H_{coup}, two preliminary general conclusions on the effect of the magnetic modulation can be drawn. First, H_{evar} leads to a time variation of the quasiparticle energies, which become $E_k(t) = E_{k,0} + \delta E_k(t)$, where

$$\delta E_k(t) = -\frac{\hbar A}{2} \sin(\omega_p t) \cos(2\theta_k) \qquad (9)$$

In the next section, we will see that the disequilibrium induced by this term and the subsequent relaxation of the condensate are at the origin the detected delayed response of the system to the driving. Second, H_{coup} represents modulation-induced interactions between the vacuum state and a doubly-excited state. Importantly, these coupling terms, which oscillate with the external frequency ω_p, are relevant only when they can induce an effective resonance between the BCS state and the two-excitation configuration, i.e., only when $\hbar\omega_p \geq 2\tilde{\Delta}_0$. The resulting heating effects will be analyzed in Section IV.

6

III. THE EFFECT OF THE QUASIPARTICLE-ENERGY VARIATION

In the regime defined by $\hbar\omega_p < 2\tilde{\Delta}_0$, the interaction terms given by H_{coup} can be discarded, and, consequently, the perturbation Hamiltonian H_{per} can be approximated as $H_{evar} = \sum_{\mathbf{k}} \delta E_k (c^{\dagger}_{\mathbf{k},\uparrow} c_{\mathbf{k},\uparrow} + c^{\dagger}_{\mathbf{k},\downarrow} c_{\mathbf{k},\downarrow})$. Hence, the driven Hamiltonian is still diagonal in the representation of the quasiparticle states of the unmodulated system. We will see that, although H_{evar} simply leads to the variation of the quasi-particle energies, the consequent effect on the gap dynamics can be quite complex; in fact, its analysis will require the generalization of our model. (Here, we will follow a treatment alternative to that presented in Ref. [14], which will allow us to simplify the characterization of the basic physics of the delay.)

The perturbation forces the system out of equilibrium: the initial thermal populations associated with the unmodulated energies do not fit the Fermi distribution $f^{eq}_k(t) = 1/(1 + e^{E_k(t)/k_B T})$ for the actual (time-varying) energies. Indeed, the gap equation now reads

$$\Delta(t) = \frac{|V_{int}|}{2} \sum_{\mathbf{k}} [2f_k(t) - 1]\sin(2\theta_k), \tag{10}$$

where $\{f_k(t)\}$ are the (changing) populations. To describe the dynamics, we must deal with the relaxation of the populations towards equilibrium, which implies extending the current Hamiltonian description. Note that a selfconsistent approach is needed. The evolving $\{E_k(t)\}$ affect the relaxation of the populations $\{f_k(t)\}$ by modifying the equilibrium distribution $\{f^{eq}_k(t)\}$. In turn, the variation of $\Delta(t)$ changes the global Hamiltonian, and, in particular, can alter the quasiparticle energies. As previously stated, in the simplified description considered here, we neglect corrections to the quasi-particle energies due to changes in the mean fields: the form given by Eq. (9) is assumed to permanently apply. (See Ref. [14] for an analysis of higher-order effects.) We will see that this simplification retains the system components responsible for the emergence of the features observed in the experiments.

A. The relaxation mechanism

The mechanism for thermalization can be assumed to be based on collisions between excited particles. Here, instead of tackling a detailed analysis of the dependence of the

relaxation on the system characteristics, we will focus on general aspects of its role in the condensate dynamics. Accordingly, we consider that the evolution of the populations is governed by the generic equation [22]

$$\frac{df_k}{dt} = -\frac{1}{\tau_f}\left[f_k(t) - f_k^{eq}(t)\right], \tag{11}$$

where $1/\tau_f$ represents the effective thermalization rate. No restrictions on the magnitude of τ_f are assumed. A similar relaxation mechanism was considered in Ref. [22] in the context of nonequilibrium superconductivity. Central to this mechanism is the idea that the relaxation is activated by the distance from the actual populations to those corresponding to the equilibrium, which are, in turn, changing as the quasiparticles energies are being modified by the driving. We have assumed a first (compact) form of characterizing that process with the introduction of the effective thermalization rate. Note that, since the system is continuously forced out of equilibrium by the driving field, i.e., the $f_k^{eq}(t)$ are permanently changing, the relaxation mechanism is always activated. The nondirect following to the modulation observed in the experiments can be anticipated to be rooted in finite values of τ_f.

Eq. (11) is an inhomogeneous linear differential equation, which is exactly solved to give

$$f_k(t) = e^{-t/\tau_f}\left(f_k(0) - \frac{1}{\tau_f}\int_0^t e^{t'/\tau_f}f_k^{eq}(t')dt'\right).$$

Furthermore, through integration by parts, we find

$$f_k(t) = e^{-t/\tau_f}\left(f_k(0) - f_k^{eq}(0)\right) + f_k^{eq}(t) - e^{-t/\tau_f}\int_0^t e^{t'/\tau_f}\frac{df_k^{eq}}{dt}(t')dt'. \tag{12}$$

This expression is simplified by taking $f_k(0) = f_k^{eq}(0)$, since the system is at equilibrium at $t = 0$. By combining Eqs. (10) and (12), we obtain the following integral-differential equation for the order parameter

$$\Delta(t) = \frac{|V_{int}|}{2}\sum_{\mathbf{k}}\left[2\left(f_k^{eq}(t) - \int_{-\infty}^t e^{-(t-t')/\tau_f}\frac{df_k^{eq}}{dt}(t')dt'\right) - 1\right]\sin(2\theta_k). \tag{13}$$

We deal now with particular regimes where we can go further in the analytical characterization of the gap evolution, and, consequently, in the identification of the delay time.

B.   The response to the modulation at small departure from equilibrium

Eq. (13) simplifies considerably in the regime defined by $E_k \sim \Delta \ll T \approx T_c$, $(k_B = 1)$. (T_c is the temperature for the BCS transition.) In this range, the approximations $f_k^{eq}(t) \simeq f_k^{eq}(0) + \frac{df_k^{eq}}{dE_k}\delta E_k(t)$ and $\frac{df_k^{eq}}{dE_k} \simeq -\frac{1}{4T_c}$ can be made [22]. In turn, we can write $\frac{df_k^{eq}}{dt} \simeq -\frac{1}{4T_c}\frac{dE_k}{dt} = \frac{\hbar A \omega_p}{8T_c}\cos(\omega_p t)\cos(2\theta_k)$. Through the incorporation of these approximations into Eq. (12), and, subsequent integration, we obtain for the populations

$$f_k(t) = f_k^{eq}(0) + \frac{\hbar A}{8T_c}\frac{\cos(2\theta_k)}{1 + (\omega_p \tau_f)^2}\left[\sin(\omega_p t) - \omega_p \tau_f\left(\cos(\omega_p t) - e^{-t/\tau_f}\right)\right]. \tag{14}$$

Then, combining this equation with Eq. (13), we find

$$\frac{\Delta(t)}{\Delta_0} = 1 + C\left[e^{-t/\tau_f}\sin\varphi + \sin(\omega_p t - \varphi)\right], \tag{15}$$

where

$$C = \frac{|V_{int}|}{\Delta_0}\frac{\hbar A}{8T_c\sqrt{1 + (\omega_p \tau_f)^2}}\sum_{\mathbf{k}}\sin(2\theta_k)\cos(2\theta_k), \tag{16}$$

and

$$\varphi = \arctan(\omega_p \tau_f).$$

Some implications of these results must be stressed:

(i) The gap evolution incorporates a transitory decay with characteristic time τ_f and a secular oscillatory behavior with frequency ω_p. The external field is not instantaneously followed: associated with the phase shift φ, there is a delay time given by $\tau_D = \frac{\varphi}{\omega_p} = \tau_f\left[1 + \mathcal{O}\left((\omega_p \tau_f)^2\right)\right]$, which can be interpreted as the condensate relaxation time. No changes in the delay are observed at different cycles of the field modulation in agreement with the experimental results. Furthermore, the detected invariance of the delay with the external frequency can be understood as associated with the small magnitude of the correction $\mathcal{O}\left((\omega_p \tau_f)^2\right)$ for the experimental conditions. The complex character of the driving mechanism is apparent in the obtained expression for the amplitude of the oscillatory term, which combines external-field parameters and characteristics of the unperturbed system. (The

factor $\sum_{\mathbf{k}} \sin(2\theta_k) \cos(2\theta_k)$ present in Eq. (16) can be standardly evaluated [22, 23].) It is worth emphasizing that the effect of the perturbation scales with the factor $A/\sqrt{1 + (\omega_p \tau_f)^2}$.

(ii) Outside the considered regime, the intricate interdependence of the gap and the populations can imply a complex nonlinear contribution of the populations to the gap relaxation [22]. Hence, one can expect that, in a general regime, the delay time can significantly differ from τ_f.

(iii) The consistency of our approach can be tested by analyzing the limits of small and large relaxation time τ_f. When τ_f is much smaller than any other characteristic time in the process, in particular, than the driving period, we find that $\varphi \to 0$ [24]. Then, there is no delay between the gap evolution and the external field. The predictions of the adiabatic approximation are consistently reproduced: for a sudden relaxation, the populations follow adiabatically, (on the relaxation time scale), the equilibrium values $\{f_k^{eq}(t)\}$ associated with the time-dependent energies. The associated gap dynamics becomes "trivial": the evolution corresponds to a sequence of equilibrium states where time enters as a parameter. On the other hand, for a very large τ_f, we obtain $C \to 0$, and, therefore, $\Delta(t) = \Delta_0$: we trivially recover that there is no change in the gap for times much smaller than the characteristic time for the evolution of the populations τ_f.

IV. HEATING EFFECTS

For $\hbar \omega_p \geq 2\tilde{\Delta}_0$, the term H_{coup} in the perturbation becomes relevant: the magnetic field can then induce an effective resonance between the fundamental state and a doubly-excited state. (See Refs. [25] and [26] for related work.) To analyze the resulting transition, we rewrite the coupling Hamiltonian as

$$H_{coup} = \hat{W} \sin(\omega_p t) \ ,$$

where

$$\hat{W} = -\frac{\hbar A}{2} \sum_{\mathbf{k}} \sin(2\theta_k) \left(c_{\mathbf{k},\uparrow}^\dagger c_{-\mathbf{k},\downarrow}^\dagger + \mathrm{h.c.} \right) .$$

Note that, because of the dependence of the factor $\sin(2\theta_k) = \left| \tilde{\Delta}_0 \right| / E_{k,0}$ on the quasiparticle energy, the coupling is less effective as the energy grows. (The opposite occurs in the

term H_{evar}, [see Eq. (7)].) The combination of this characteristic with the form of the density of states will be shown to determine prominent features of the system response. The transfer from the BCS state $|\Psi_{BCS}\rangle$ to the doubly-excited state $|f\rangle = c_{\mathbf{k},\uparrow}^{\dagger} c_{-\mathbf{k},\downarrow}^{\dagger} |\Psi_{BCS}\rangle$ of the continuum of quasi-particle states can be evaluated applying Fermi's Golden Rule. (The reverse process, i.e., the decay of doubly-excited states induced by H_{coup}, can be neglected: given the range of temperatures considered, the population of excited states is always much smaller than that of the fundamental state. Also relevant to the lack of symmetry in the reverse transition is the continuum structure of the excited states.) As corresponds to a sinusoidal perturbation, we have for the transition rate:

$$
\begin{aligned}
\gamma(\omega_p) &= \frac{\pi}{2\hbar} \sum_f \left| \left\langle f \left| \hat{W} \right| \Psi_{BCS} \right\rangle \right|^2 \delta(E_f - E_{BCS} - \hbar\omega_p) \\
&= \frac{\pi\hbar}{8} A^2 \sum_{\mathbf{k}} \sin^2(2\theta_k) \delta(2E_{k,0} - \hbar\omega_p).
\end{aligned}
\tag{17}
$$

The sum in $\mathbf{k}$ is standardly converted into an integral: $\sum_{\mathbf{k}} \to \frac{\mathcal{V}}{(2\pi)^3} \int d\mathbf{k}$ [27]. [Here, $\mathcal{V}$ denotes the quantization volume, which disappears in the final expression as a scaling with $\mathcal{V}^{-1/2}$ is incorporated into the definition of the operators introduced in Eq. (1)]. The integral is evaluated by changing to the variable $E_{k,0}$. Indeed, incorporating the density of states obtained from the dispersion relation given by Eq. (4), the transition rate is found to be given by

$$
\begin{aligned}
\gamma(\omega_p) &= \frac{1}{2\pi} \left(\frac{m}{2\hbar^2} \right)^{3/2} \tilde{\Delta}_0^2 \tilde{A}^2 \frac{1}{\omega_p} \left[\sqrt{(\hbar\omega_p/2)^2 - \tilde{\Delta}_0^2} + \mu - V_{int} n_0 \right]^{1/2} \times \\
&\quad \left[(\hbar\omega_p/2)^2 - \tilde{\Delta}_0^2 \right]^{-1/2} \Theta(\hbar\omega_p - 2\tilde{\Delta}_0),
\end{aligned}
\tag{18}
$$

where $\tilde{A} = A\mathcal{V}^{1/2}$, and $\Theta(x)$ is the Heaviside step function: the excitation process is activated only for frequencies equal or larger than the threshold value $\hbar\omega_p^{th} = 2\tilde{\Delta}_0$. Note that the divergence at the threshold is a consequence of the singularity of the density of states in the BCS model at $E_{k,0} = \tilde{\Delta}_0$. It is important to take into account that this failure of the perturbative scheme does not invalidate the identification of the physical mechanism responsible for heating: the fast decrease of the transition rate with ω_p allows the applicability of the used approach sufficiently far from the threshold.

The transfer of population from the fundamental state to the excited states implies the decrease of $\Delta(t)$. This is apparent from Eq. (10): although the total population is conserved in the transition, $\Delta(t)$ diminishes since the factor $\sin(2\theta_k)$ decreases as $E_{k,0}$ grows. Since the excitation is permanently activated by the driving, the resulting damping process is continuous, in agreement with the character of the detected decay of the condensate fraction. The global picture of the gap dynamics that emerges from combining this effect with the delayed oscillation analyzed in Sec. III corresponds to the behavior detected in the experiments. Note that, in the applied approach, the mechanisms for delay and damping can be considered to work in parallel. In this sense, it is worth pointing out that the term of population gain for the excited states that, because of heating, should be added to Eq. (11) is irrelevant given the small contribution of those high-level populations to the gap dynamics.

The applicability of the perturbative approach can be assessed from the analysis of the dependence of the system output on the modulation parameters. Both, the damping coefficient γ and the amplitude of the oscillatory component C, given by Eq. (16), diminish for decreasing modulation amplitudes and growing frequencies. The observation of oscillatory behavior along with damping requires working with decay times larger than the driving period. Given the dependence of γ on A^2, and the requirement $\hbar\omega_p^{th} \geq 2\tilde{\Delta}_0$ for the emergence of damping, that situation can occur for sufficiently small modulation amplitudes and large frequencies. Then, it seems possible to reproduce that situation in a range of parameters where the applicability of the perturbative scheme can be guaranteed. Technical details of the measurement of the condensate fraction, which is the magnitude reproduced by our model, can be found in Ref. [15].

We have assumed that the trapping conditions implemented in the referred experiments do not crucially affect the main characteristics of the observed features. Indeed, as our uniform description qualitatively reproduces the experimental results, the robustness of the identified physical mechanisms against spatial non-uniformities can be conjectured. For a smooth external potential $U(\vec{r})$, which corresponds to the practical conditions, a local-density approximation can be applied to generalize the uniform picture. Accordingly, trapping can be incorporated in the previous framework by replacing the chemical potential as $\mu \to \mu(\vec{r}) = \mu - U(\vec{r})$ [27]. The consequent use of local fields implies less compact results but does not affect the basic physics underlying the observed features. One of the effects of nonuniformity on heating seems evident: the local fields $\Delta_0(\vec{r})$, $\phi_{m,0}(\vec{r})$, $n_0(\vec{r})$, and, in

turn, $\tilde{\Delta}_0(\vec{r})$, can be expected to smoothen the sharp threshold for the onset of excitations. Although the previously derived expressions can be modified by the averaging over the distribution along the trap, the former basic picture still applies.

V. CONCLUDING REMARKS

The considered variation of the coupled fermion-boson model has been shown to give useful clues to understanding the dynamics of atomic fermion pairs with modulated interaction strength. The perturbative scheme set up from the basic BCS approach has allowed isolating the role of the different elements of the system. The emergence of specific dynamical features has been found to depend on the time scales of the system components: the relative magnitudes of the driving period, the inverse gap frequency, and the relaxation time determine the characteristics of the system response. The interest of further experimental work on specific aspects of the modulation scheme is evident. Particularly valuable can be the experimental characterization of the system response for driving frequencies slightly larger than the threshold, where the perturbative approach fails. Also interesting can be checking the inhibition of damping for frequencies smaller than $2\tilde{\Delta}_0$.

It is of interest to mention recent related works on alternative approaches to the study of similar systems. In this sense, it is pertinent to point out the advances in the characterization of the system parameters which are reported in Ref. [31] and the applications of analogue methodology to p-wave interacting Fermi gases [32]. Also valuable is to establish a parallelism between the considered scenario and similar modulation techniques applied in bosonic systems. (Ref. [33] presents interesting experimental findings on the production of ultracold molecules via a sinusoidal modulation of the magnetic field. Subsequent theoretical analysis was presented in Ref. [34].)

Finally, it is worth pointing out that the applicability of the study is not restricted to the field of ultracold atomic gases. The central issue in the analysis, namely, the gap dynamics [28], in particular, the effect of changes in the populations on the evolution of the condensate fraction, is relevant to topics ranging from nonequilibrium superconductivity [22, 29] to quenched dynamics in superfluid ^3He [30]. In this sense, the analytical character of the study can be particularly useful given the difficulty of dealing with out-of-equilibrium

situations in those contexts.

[1] For recent reviews, see S. Giorgini, L.P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **80**, 1215 (2008); I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008); M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen De, and U. Sen, Adv. Phys. **56**, 243 (2007); and references therein.

[2] M. Greiner *et al.*, Nature (London) **426**, 537 (2003).

[3] S. Jochim *et al.*, Science **302**, 2101 (2003).

[4] M. W. Zwierlein *et al.*, Phys. Rev. Lett. **91**, 250401 (2003).

[5] T. Bourdel *et al.*, Phys. Rev. Lett. **93**, 050401 (2004).

[6] M. Bartenstein *et al.*, Phys. Rev. Lett. **92**, 120401 (2004).

[7] C. A. Regal *et al.*, Phys. Rev. Lett. **92**, 040403 (2004).

[8] M. W. Zwierlein *et al.*, Phys. Rev. Lett. **92**, 120403 (2004).

[9] K. E. Strecker *et al.*, Phys. Rev. Lett. **91**, 080406 (2003).

[10] M. Bartenstein *et al.*, Phys. Rev. Lett. **92**, 203201 (2004).

[11] J. Kinast *et al.*, Phys. Rev. Lett. **92**, 150402 (2004).

[12] W. Ketterle and M. W. Zwierlein, Proceedings of the International School of Physics "Enrico Fermi", Course CLXIV, (IOS Press, Amsterdam, 2008).

[13] E. Taylor *et al.*, Phys. Rev. A **74**, 063626 (2006); N. Fukushima *et al.*, Phys. Rev. A **75**, 033609 (2007).

[14] J. Plata, Europhys. Lett. **87**, 50001 (2009).

[15] M. W. Zwierlein *et al.*, Phys. Rev. Lett. **94**, 180401 (2005).

[16] M. Holland *et al.*, Phys. Rev. Lett. **87**, 120406 (2001).

[17] E. Timmermans, K. Furuya, P. W. Milonni, and A. K. Kerman, Phys. Lett. A **285**, 228 (2001).

[18] M. L. Chiofalo *et al.*, Phys. Rev. Lett. **88**, 090402 (2002).

[19] Y. Ohashi and A. Griffin, Phys. Rev. Lett. **89**, 130402 (2002).

[20] Y. Ohashi and A. Griffin, Phys. Rev. A **72**, 013601 (2005).

[21] Y. Ohashi and A. Griffin, Phys. Rev. A **72**, 063606 (2005).

[22] M. Tinkham, *Introduction to Superconductivity* (Dover Publications, New York, 1996).

[23] M. Marini *et al.*, Eur. Phys. J. B **1**, 151 (1998).

[24] W. Yi and L. M. Duan, Rev. A **73**, 013609 (2006)

[25] M. Greiner *et al.*, Phys. Rev. Lett. **94**, 070403 (2005).

[26] J. Plata, Phys. Rev. A **74**, 013603 (2006).

[27] L. Salasnich, N. Manini, and A. Parola, Phys. Rev. A **72**, 023621 (2005).

[28] R. G. Scott, F. Dalfovo, L. P. Pitaevskii, and S. Stringari, Phys. Rev. A **86**, 053604 (2012).

[29] J.A. Schmid and G. Schon, J. Low Temp. Phys. **20**, 207 (1975).

[30] G. L. Warner and A. J. Leggett, Phys. Rev B **71**, 134514 (2005).

[31] M. H. Szymańska *et al.*, Phys. Rev. A **72**, 013610 (2005).

[32] L. Austen *et al.*, Phys. Rev. A **87**, 023610 (2013).

[33] S. T. Thomson *et al.*, Phys. Rev. Lett. **95**, 190404 (2005).

[34] T. M. Hanna *et al.*, Phys. Rev. A **75**, 013606 (2007).